

Name: _____

Solutions

Work in groups to answer as many problems as you can. Ask questions if you get stuck. The numbers used on this worksheet may require a calculator. Keep in mind that numbers you will have on exams will be nice enough to do without a calculator.

1. Complete the square on the following functions.

(a) $f(x) = x^2 - 2x - 3$

$$= (x-1)^2 + d$$

$$= x^2 - 2x + 1 + d$$

$$\Rightarrow d = -4$$

Answer: $(x-1)^2 - 4$

(d) $f(x) = x^2 + 4x - 24$

$$= (x+2)^2 + d$$

$$= x^2 + 4x + 4 + d$$

$$\Rightarrow d = -28$$

Answer: $(x+2)^2 - 28$

(b) $f(x) = x^2 + 4x$

$$= (x+2)^2 + d$$

$$= x^2 + 4x + 4 + d$$

$$\Rightarrow d = -4$$

Answer: $(x+2)^2 - 4$

(e) $f(x) = x^2 - x$

$$= (x - \frac{1}{2})^2 + d$$

$$= x^2 - x + \frac{1}{4} + d$$

$$\Rightarrow d = -\frac{1}{4}$$

Answer: $(x - \frac{1}{2})^2 - \frac{1}{4}$

(c) $f(x) = x^2 - 6x$

$$= (x-3)^2 + d$$

$$= x^2 - 6x + 9 + d$$

$$\Rightarrow d = -9$$

Answer: $(x-3)^2 - 9$

(f) $f(x) = -4x^2 + 8$

$$= -4(x^2 - 2)$$

$$= -4((x+0)^2 + d)$$

$$= -4(x^2 + d)$$

$$\Rightarrow d = -2$$

Answer: $-4(x^2 - 2)$

2. Write each of the following quadratics in "standard form".

(a) $f(x) = 2x^2 - 8x + 16$

$$\begin{aligned} &= 2(x^2 - 4x) + 16 \\ &= 2((x-2)^2 - 4) + 16 \\ &= 2(x-2)^2 + 8 \end{aligned}$$

Answer: $2(x-2)^2 + 8$

(d) $f(x) = 8x^2 + 2$

$$\begin{aligned} &= 8(x^2) + 2 \\ &= 8((x+0)^2 - 0) + 2 \\ &= 8x^2 + 2 \end{aligned}$$

Answer: $8x^2 + 2$

(b) $f(x) = -x^2 - 4x - 3$

$$\begin{aligned} &= -(x^2 + 4x) - 3 \\ &= -((x+2)^2 - 4) - 3 \\ &= -(x+2)^2 + 1 \end{aligned}$$

Answer: $-(x+2)^2 + 1$

(e) $f(x) = 4x^2 - 8x + 7$

$$\begin{aligned} &= 4(x^2 - 2x) + 7 \\ &= 4((x-1)^2 - 1) + 7 \\ &= 4(x-1)^2 + 3 \end{aligned}$$

Answer: $4(x-1)^2 + 3$

(c) $f(x) = 3x^2 + 12x - 1$

$$\begin{aligned} &= 3(x^2 + 12x) - 1 \\ &= 3((x+6)^2 - 36) - 1 \\ &= 3(x+6)^2 - 109 \end{aligned}$$

Answer: $3(x+6)^2 - 109$

(f) $f(x) = x^2 + 4$

$$\begin{aligned} &= 1(x^2) + 4 \\ &= 1((x+0)^2 - 0) + 4 \\ &= x^2 + 4 \end{aligned}$$

Answer: $x^2 + 4$

3. Write each of the following quadratic functions in "standard form." Then fill out the table and plot the graph, labeling the y intercept, roots and the vertex.

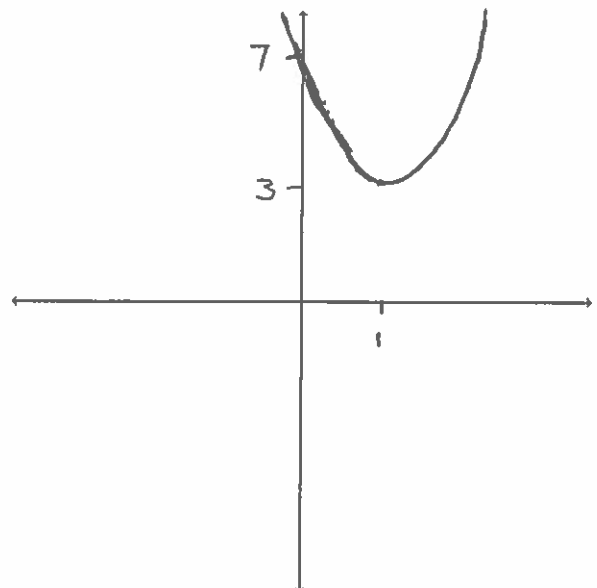
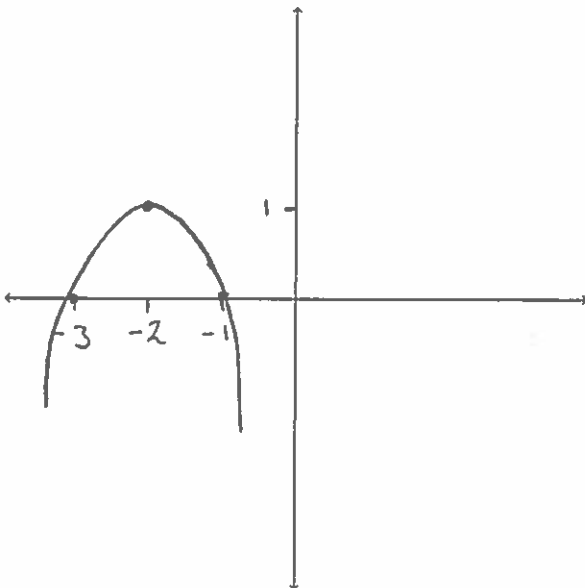
(a) $f(x) = -x^2 - 4x - 3 = -(x+1)(x+3)$
 $= -(x^2 + 4x) - 3$
 $= -((x+2)^2 - 4) - 3$
 $= -(x+2)^2 + 1$

(b) $f(x) = 4x^2 - 8x + 7$
 $= 4(x^2 - 2x) + 7$
 $= 4((x-1)^2 - 1) + 7$
 $= 4(x-1)^2 + 3$

min = 3 \Rightarrow no roots

Domain:	$(-\infty, \infty)$	y-intercept:	-3
Range:	$(-\infty, 1]$	Minimum:	N/A
Maximum:	$(-2, 1)$	Increasing:	$(-\infty, -2)$
Decreasing:	$(-2, \infty)$	Roots:	-1, -3

Domain:	$(-\infty, \infty)$	y-intercept:	7
Range:	$[3, \infty)$	Minimum:	(1, 3)
Maximum:	N/A	Increasing:	$(1, \infty)$
Decreasing:	$(-\infty, 1)$	Roots:	N/A



(c) $f(x) = x^2 + 4$

$$= x^2 + 4$$

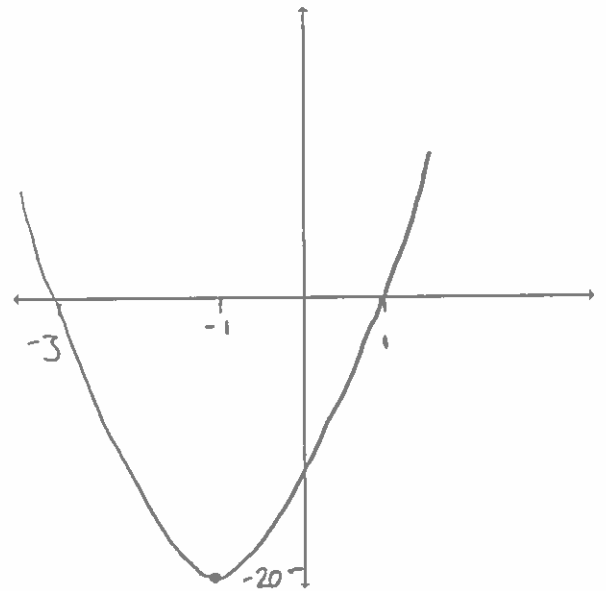
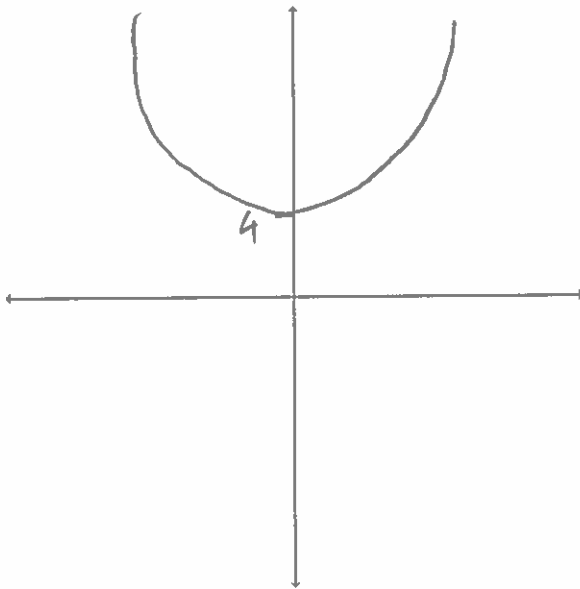
min = 4 \Rightarrow no roots

(d) $f(x) = 5x^2 + 10x - 15 = 5(x-1)(x+3)$

$$\begin{aligned}
 &= 5(x^2 + 2x) - 15 \\
 &= 5((x+1)^2 - 1) - 15 \\
 &= 5(x+1)^2 - 20
 \end{aligned}$$

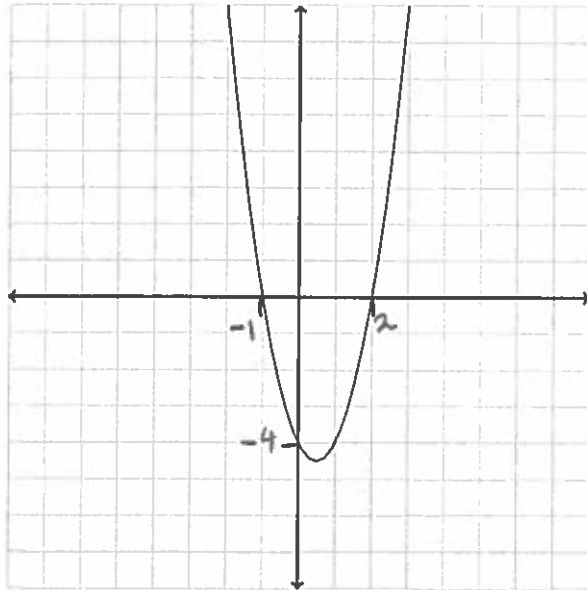
Domain:	$(-\infty, \infty)$	y-intercept:	4
Range:	$[4, \infty)$	Minimum:	(0, 4)
Maximum:	N/A	Increasing:	$(0, \infty)$
Decreasing:	$(-\infty, 0)$	Roots:	N/A

Domain:	$(-\infty, \infty)$	y-intercept:	-15
Range:	$[-20, \infty)$	Minimum:	(-1, -20)
Maximum:	N/A	Increasing:	$(-1, \infty)$
Decreasing:	$(-\infty, -1)$	Roots:	1, -3



4. For each of the given graphs, determine which function it represents. Each square represents 1 unit.

(a)

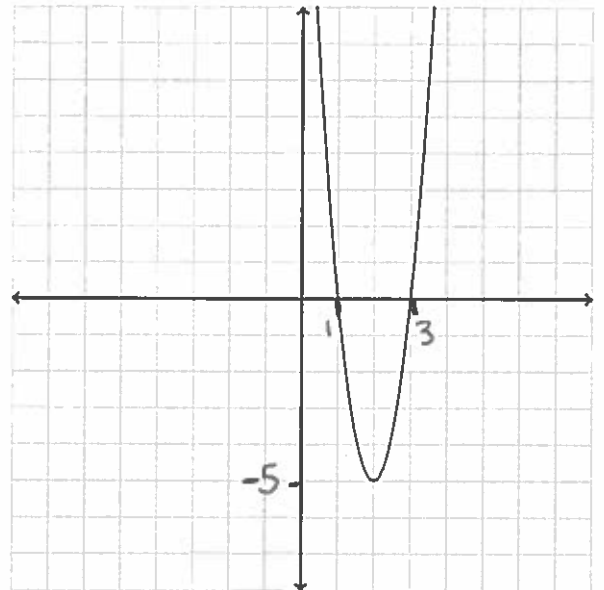


roots $-1, 2 \Rightarrow f(x) = a(x+1)(x-2)$

$a(x+1)(x-2) = a(x^2 - x - 2)$

$y \text{ int} = -4 \Rightarrow a = 2 \Rightarrow \boxed{f(x) = 2(x+1)(x-2)}$

(c)



roots $= 1, 3 \Rightarrow f(x) = a(x-1)(x-3)$

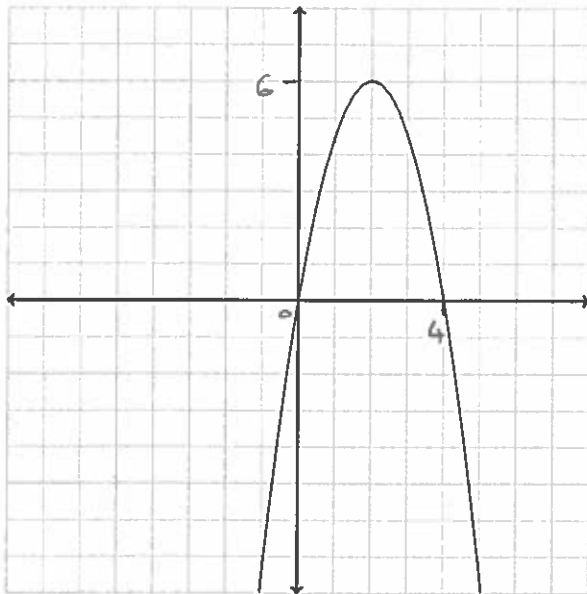
$a(x-1)(x-3) = a(x^2 - 4x + 3)$

$= a(x-2)^2 - a$

vertex $\Rightarrow -a = -5 \Rightarrow a = 5$

$\boxed{f(x) = 5(x-1)(x-3)}$

(b)



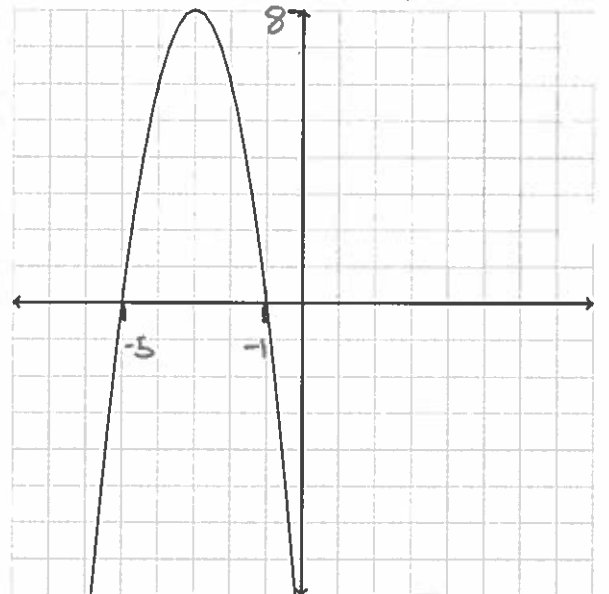
roots $0, 4 \Rightarrow f(x) = ax(x-4)$

$ax(x-4) = a(x^2 - 4x) = a(x-2)^2 - 4a$

vertex $\Rightarrow -4a = 6 \Rightarrow a = -3/2$

$\boxed{f(x) = -3/2 x(x-4)}$

(d)



roots $= -1, -5 \Rightarrow f(x) = a(x+1)(x+5)$

$a(x+1)(x+5) = a(x^2 + 6x + 5)$

$= a(x+3)^2 - 4a$

vertex $\Rightarrow -4a = 8 \Rightarrow a = -2$

$\boxed{f(x) = -2(x+1)(x+5)}$

Cont.

5. Describe each of the following transformations of $f(x)$, in words. Be careful which order you write the transformations in.

(a) $f(x-2)$

Shift right 2 units

(e) $f(3x) - 6$

Stretch in x by s.f $\frac{1}{3}$,
then shift down 6 units

(b) $5f(x-2)$

shift right 2 units,
then stretch in y by
s.f 5.

(f) $f(-x) - 1$

reflect in y , then
shift down 1 unit

(c) $f(x) - 3$

shift down 3 units

(g) $2f(3x)$

stretch in x by s.f $\frac{1}{3}$,
then stretch in y by
s.f 2

(d) $f(x-3) + 3$

shift right 3 units,
then shift up 3 units.

(h) $-f(x) - 7$

Reflect in x , then
shift down 7 units.